INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.3-2.4]

FUNCTIONS

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

FUNCTIONS AND GRAPHS - USE OF GDC

1. [Maximum mark: 7]

Consider the function $f(x) = 3x + 7$

(a) Find $f(0)$ and $f(2)$ [2]

(b) Write down $f(a)$ in terms a. [1]

(c) Find a, given that $f(a) = 10$ [1]

(d) Find b, given that $f(b) = 22$ [1]

(e) Hence, complete the following table of values [2]

x	0			4	5
$f(x)$		10	13	19	

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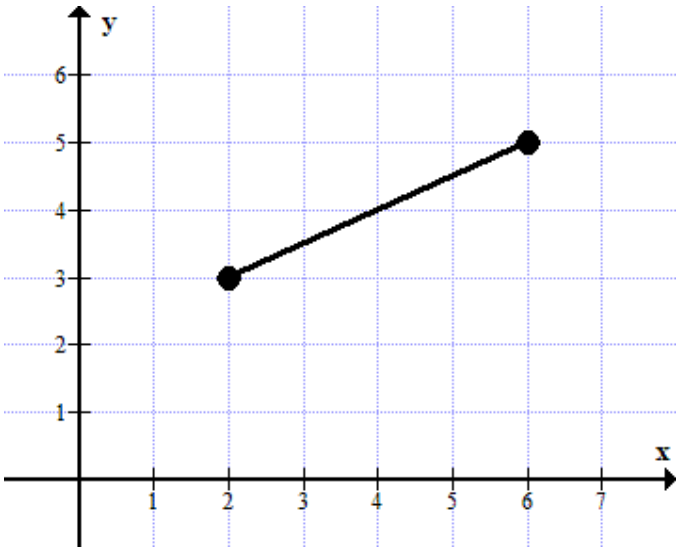
2. [Maximum mark: 7]

The graph of the function f is shown below.



(a) Write down the domain of the function. [1]

(b) Write down the range of the function. [1]

(c) Write down the values of (i) $f(2)$, (ii) $f(4)$ [2]

(d) Given the $f(a)=5$, write down the value of a [1]

(e) Write down the minimum value and the maximum value of the function. [2]

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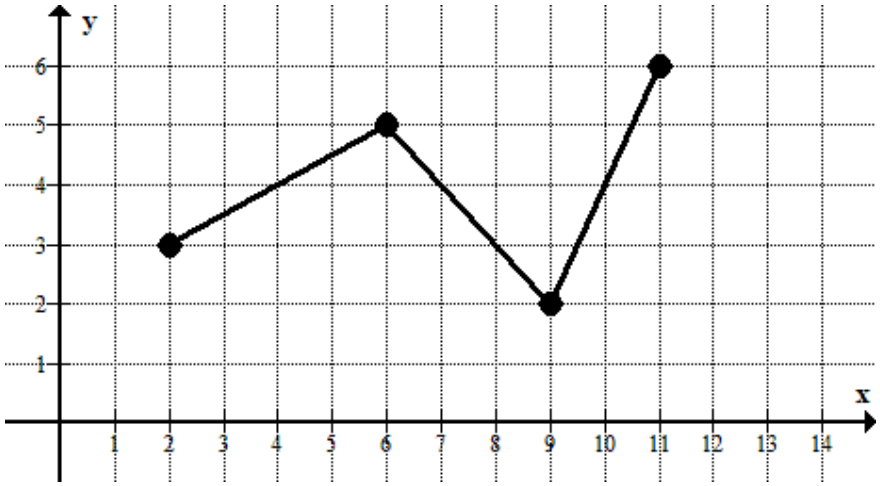
3. [Maximum mark: 10]

The graph of the function f is shown below.



(a) Write down the domain of the function. [1]

(b) Write down the range of the function. [1]

(c) Write down the minimum value and the maximum value of the function. [2]

(d) Complete the following table of values [4]

x	2	4	6		
$f(x)$				2	6

(e) Given that $f(x) = 4$, there are three possible values for x. Write down the possible values of x. [2]

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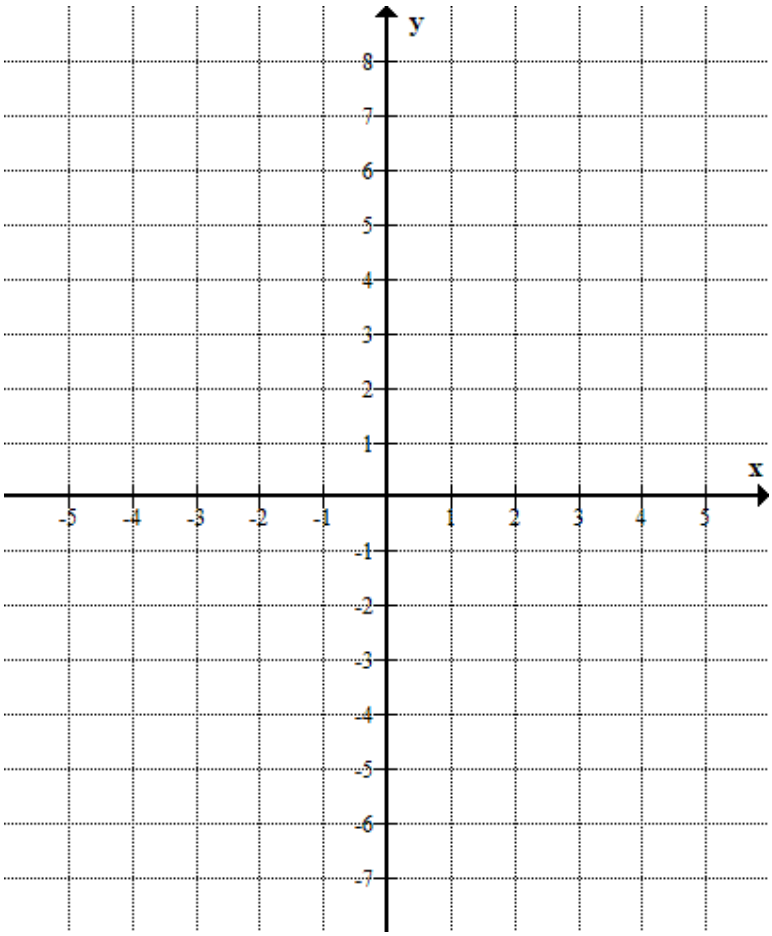
4. [Maximum mark: 8]

(a) On the same set of axes, sketch the graphs of the following two functions

$$f(x) = 2x, \qquad -3 \le x \le 3$$
$$g(x) = 2x + 2, \qquad -3 \le x \le 3$$



[3]

(b) Write down the range of each function (for the given domain).

Function	$f(x) = 2x$	$g(x) = 2x + 2$
Range		

[2]

(c) The point points $\mathrm{P}(a,4)$ and $\mathrm{Q}(2,b)$, lie on the graph of g.

(i) write down the values of a and b.

(ii) Indicate the points P and Q on the diagram above. [3]

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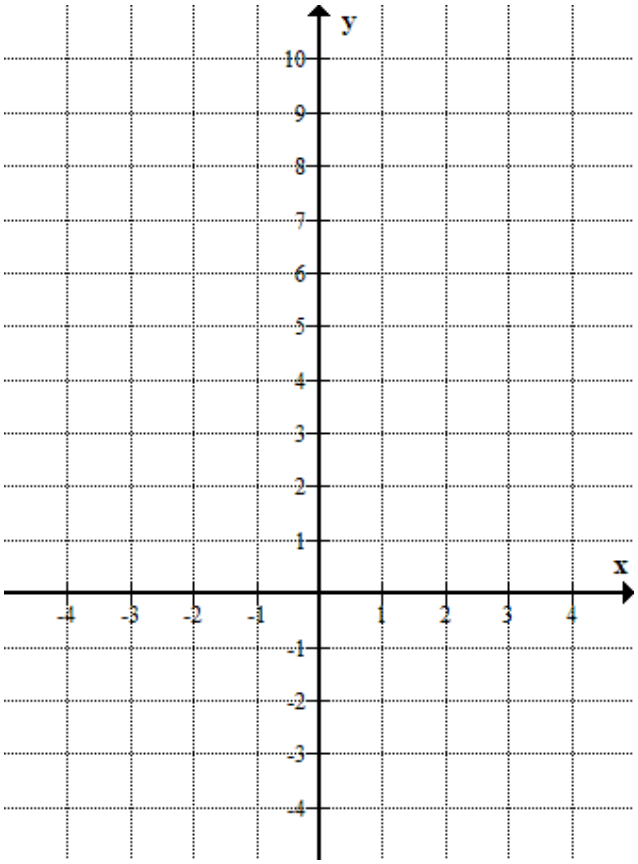
5. [Maximum mark: 10]

(a) On the same diagram below, sketch the graphs of the following two functions

$$f(x) = x^2, \qquad -3 \le x \le 3$$
$$g(x) = x^2 - 2, \qquad -3 \le x \le 3$$ [3]



For each function, write down

(b) the range (for the given domain); [3]

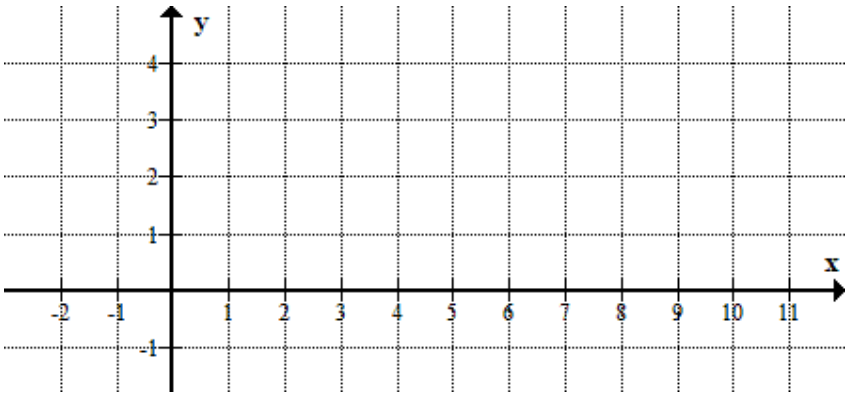
(c) the intersection points with the line $y = x$. [2]

(d) the intersection points with the horizontal line $y = 1$. [2]

Function	Range	Points of intersection with line $y = x$	Points of intersection with line $y = 1$
$y = x^2$			
$y = x^2 - 2$			

6. [Maximum mark: 5]

(a) Sketch the graph of the function $f(x) = \sqrt{x}$, $0 \le x \le 9$.



[2]

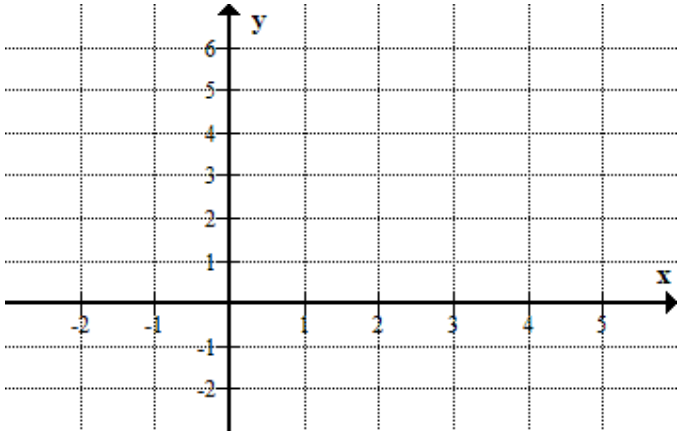
(b) Complete the following table

Range of f	
Point of intersection with the line $y = 1$	
Point of Intersection with the line $y = 2$	

[3]

7. [Maximum mark: 6]

(a) Sketch the graph of the function $f(x) = 3x - 3\sqrt{x}$, $0 \le x \le 4$.



[3]

(b) Complete the following table

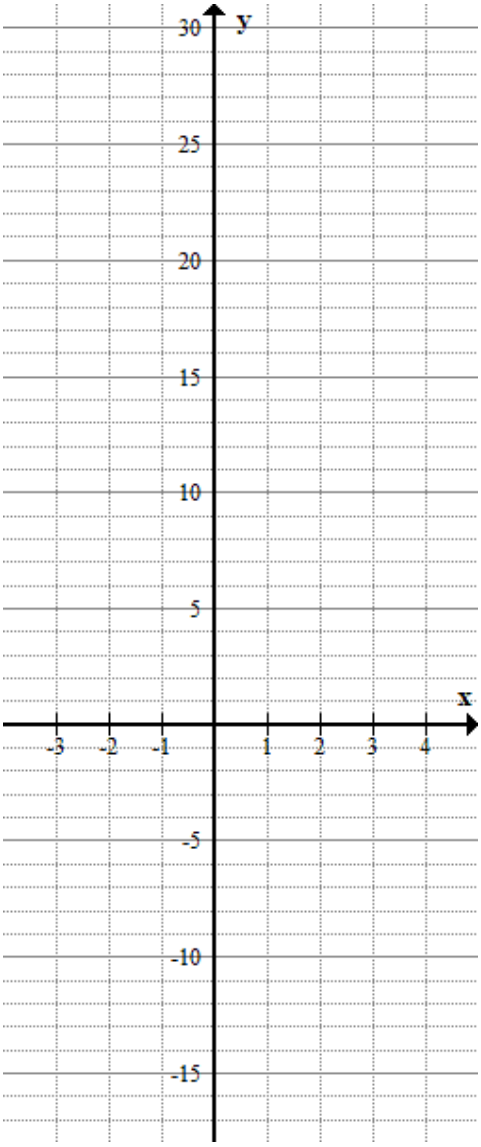
Coordinates of end points	
Coordinates of minimum point	
Range	

[3]

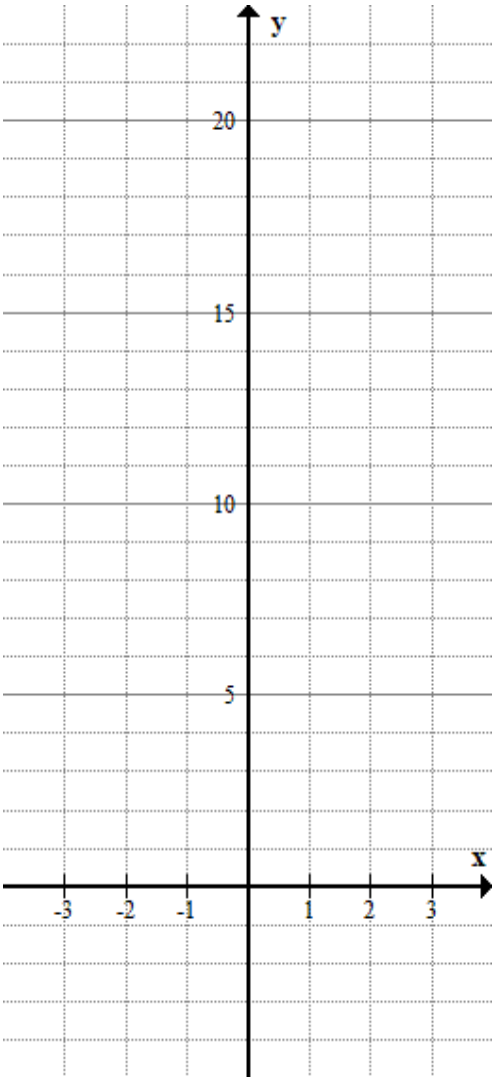
8. [Maximum mark: 10]

(a) Sketch the graphs of the following two functions.

$f(x) = x^3$



$g(x) = x^4$



(b) Write down the largest possible domain and the corresponding range.

D_f:	R_f:

D_g:	R_g:

[6]

(c) Write down the range of these functions under the following restrictions

$f(x) = x^3$

D_f: $1 \le x \le 2$	R_f:
D_f: $-1 \le x \le 2$	R_f:

$g(x) = x^4$

D_g: $1 \le x \le 2$	R_g:
D_g: $-1 \le x \le 2$	R_g:

[4]

9. [Maximum mark: 10]

Let

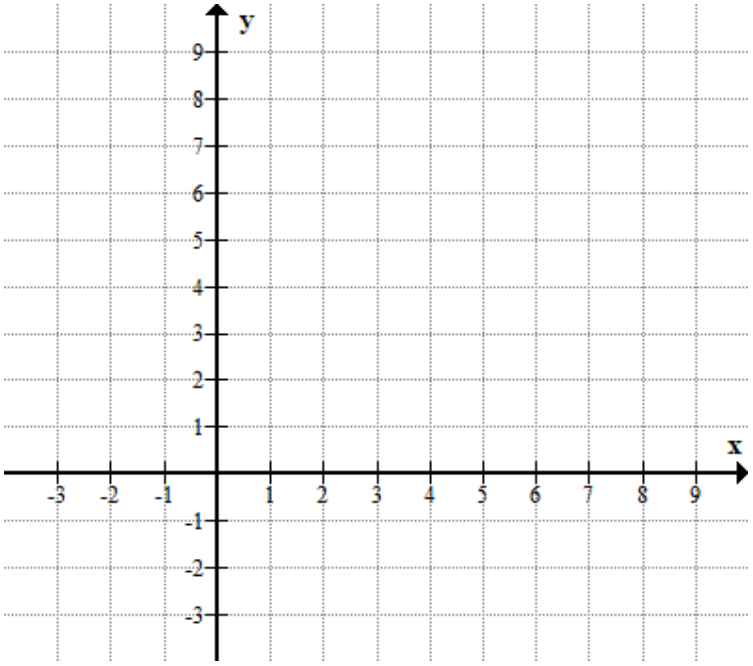
$$f(x)=\begin{cases} x-2 & 0\le x<5 \\ 5x-22 & 5\le x\le 6 \end{cases}$$

(a) Complete the table of values

x	0	1	2	3	4	5	6
$f(x)$							

[3]

(b) Sketch the graphs of the function on the diagram below



[3]

(c) Write down the domain and the range of the function. [2]

(d) Write down the y - intercept and the x - intercept. [2]

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10. [Maximum mark: 6]

Find the largest possible domain for the following functions

$f(x) = 3 - 6x$ $\quad$ $g(x) = \frac{7}{3 - 6x}$ $\quad$ $h(x) = \sqrt{3 - 6x}$ $\quad$ $k(x) = \log(3 - 6x)$

11. [Maximum mark: 8]

Find the largest possible domain for the following functions

$f(x) = x^2 - 4$ $\quad$ $g(x) = \frac{7}{x^2 - 4}$ $\quad$ $h(x) = \sqrt{x^2 - 4}$ $\quad$ $k(x) = \log(x^2 - 4)$

12. [Maximum mark: 6]

The function f is given by $f(x) = \sqrt{x-4} + \frac{2}{x-5} + \log(10-x)$.

Find the largest possible domain of the function.

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13. [Maximum mark: 5]

The function f is given by $f(x) = \sqrt{x+7} + 5$

(a) Write down the domain of the function. [2]

(b) Sketch the graph of the function (by using the GDC) to obtain the range of the function. [3]

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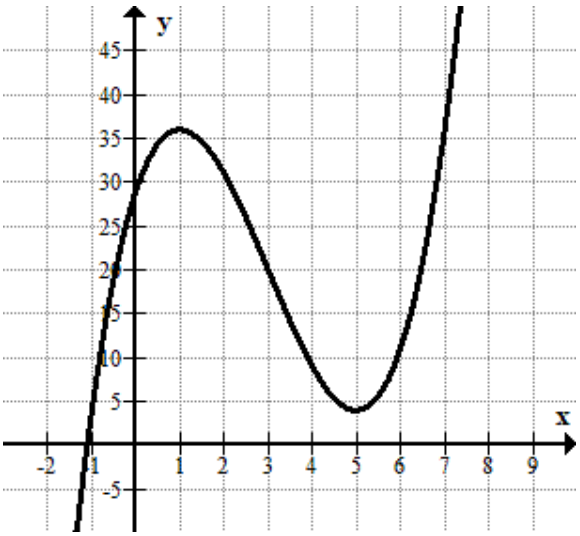
14. [Maximum mark: 8]

Find the range for following functions in the given domain

$f(x)=2x^2-8x+9, \quad x\in R$	
$f(x)=2x^2-8x+9, \quad 0\le x\le 2$	
$f(x)=2x^2-8x+9, \quad 0\le x\le 5$	
$f(x)=2x^2-8x+9, \quad x\le 0$	

15. [Maximum mark: 12]

Part of the graph of the function $f(x)=x^3-9x^2+15x+29$ is shown below



Find the range for following functions in the given domain

$f(x)=x^3-9x^2+15x+29, \quad x\in R$	
$f(x)=x^3-9x^2+15x+29, \quad x\ge 0$	
$f(x)=x^3-9x^2+15x+29, \quad x\le 0$	
$f(x)=x^3-9x^2+15x+29, \quad 0\le x\le 6$	
$f(x)=x^3-9x^2+15x+29, \quad 0\le x\le 7$	
$f(x)=x^3-9x^2+15x+29, \quad 0\le x\le 8$	

16. [Maximum mark: 5]

Let $f(x) = x + 5$

(a) Complete the following table for the function f and the inverse function f^{-1}

x	$f(x)$
0	
5	
10	
12	
20	

x	$f^{-1}(x)$
5	0
	5
	10
17	
	20

[4]

(b) Write down the equation of the inverse function $f^{-1}(x)$ in terms of x. [2]

(c) Given that $P = Q + 5$, write down Q in terms of P [1]

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17. [Maximum mark: 5]

(a) Given that $P = 2Q + 5$, write down Q in terms of P. [2]

(b) Write down the equation of the inverse function of $f(x) = 2x + 5$ [2]

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18. [Maximum mark: 6]

Complete the following table with the inverse of each function

Original function	Inverse function
$f(x)=x+5$	$f^{-1}(x)=x-5$
$f(x)=x-5$	
$f(x)=x+100$	
$f(x)=3x$	
$f(x)=\frac{x}{5}$	
$f(x)=x^3$	
$f(x)=3x+100$	

19. [Maximum mark: 4]

Consider the following table of values for f

x	1	2	3	4	5	6
$f(x)$	3	4	5	6	1	2

(a) Write down the values of

(i) $f(1)$ (ii) $f^{-1}(1)$ [2]

(b) Given that $f(x)=2$ write down the value of x. [1]

(c) Given that $f^{-1}(x)=2$ write down the value of x. [1]

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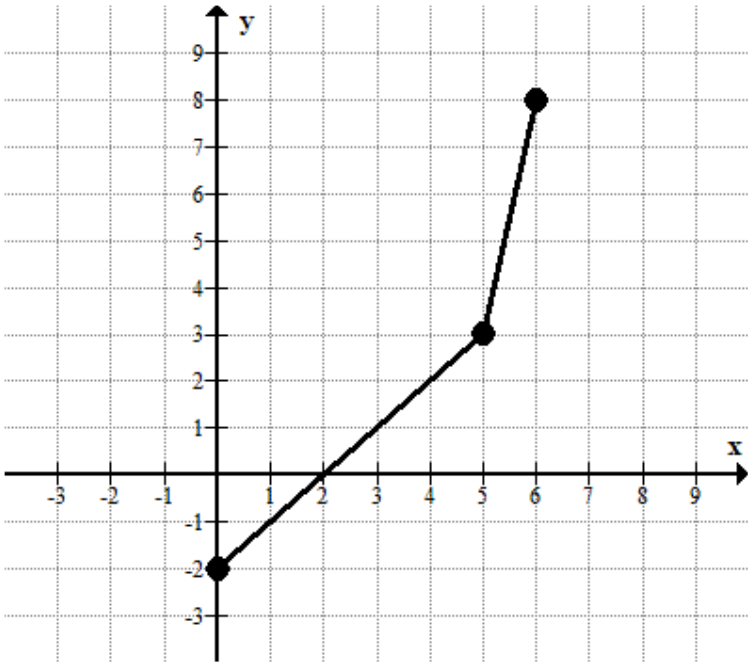
20. [Maximum mark: 12]

The graph of the function f is shown below.



(a) Write down the values of

(i) $f(0)$ (i) $f(2)$ (ii) $f(4)$ [3]

(b) Write down the values of

(i) $f^{-1}(3)$ (ii) $f^{-1}(8)$ (iii) $f^{-1}(-1)$ (iv) $f^{-1}(0)$ [4]

(c) Write down the solution of the equation $f(x) = 0$ [1]

(d) Write down the solution of the equation $f(x) = 2$ [1]

(e) On the diagram above sketch the graph of f^{-1} [3]

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21. [Maximum mark: 10]

The function f is given by $f(x) = x^2$, for $0 \le x \le 3$.

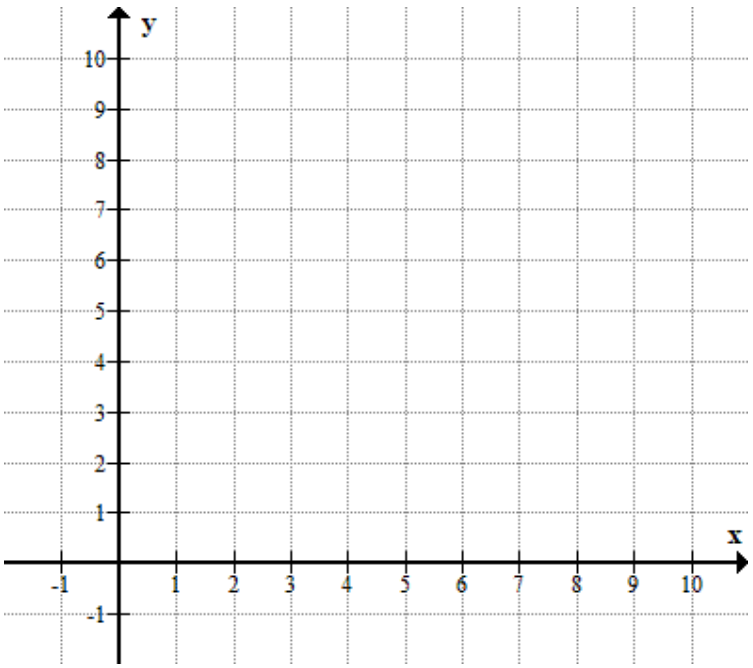
(a) Sketch the graph of f. [2]

(b) State the domain and the range of f. [2]

(c) Find the inverse function f^{-1}. [2]

(d) On the same axes with f, sketch the graph of f^{-1} [2]

(e) State the domain and the range of f^{-1}. [2]



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PRACTICE ON THE GDC

Note: Most probably the following functions are known yet:

$$y = \sin x, \quad y = \cos x, \quad y = \log x, \quad y = \ln x, \quad y = 2^x, \quad y = e^x$$

We are going to study them in due course. However, you may use your GDC to see how their graphs look like.

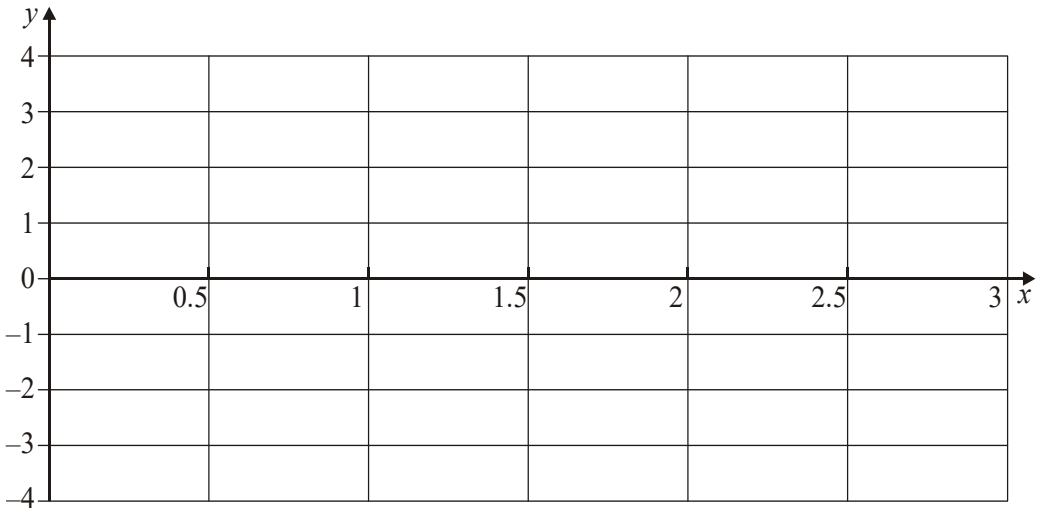
For the trigonometric functions $y = \sin x,\ y = \cos x$ make sure that x is in **radians** in your GDC.

Use your GDC to answer the following questions

22. [Maximum mark: 8]

Let $f(x) = 2 + \cos(2x) - 2\sin(0.5x)$ for $0 \le x \le 3$, where x is in **radians**.

(a) On the grid below, sketch the curve of $y = f(x)$, indicating clearly the point P on the curve where the function has a max or min value. [4]



(b) Write down the solutions of $f(x) = 0$. [2]

(c) Write down the range of f. [2]

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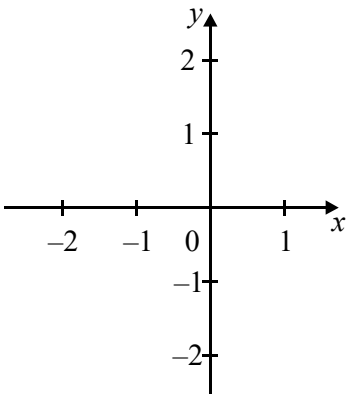
23. [Maximum mark: 10]

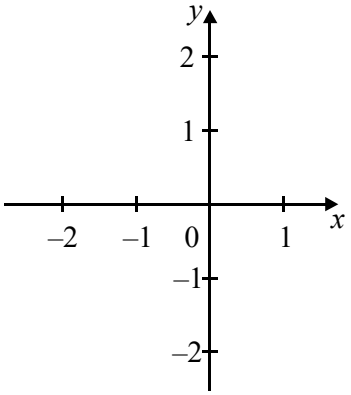
(a) On the following diagram, sketch the graphs of $y = 2^x$ and $y = \cos x$ for $-2 \leq x \leq 1$.



[3]

(b) The equation $2^x = \cos x$ has a solution between –2 and –1. Find this solution. [2]

(c) On the following diagram, sketch the graph of $g(x) = 2^x - \cos x$, for $-2 \leq x \leq 1$. [2]



(d) Write down the x-intercept of the function $g(x) = 2^x - \cos x$, between –2 and –1. [1]

(e) Solve the inequality $2^x < \cos x$ for $-2 \leq x \leq 1$. [2]

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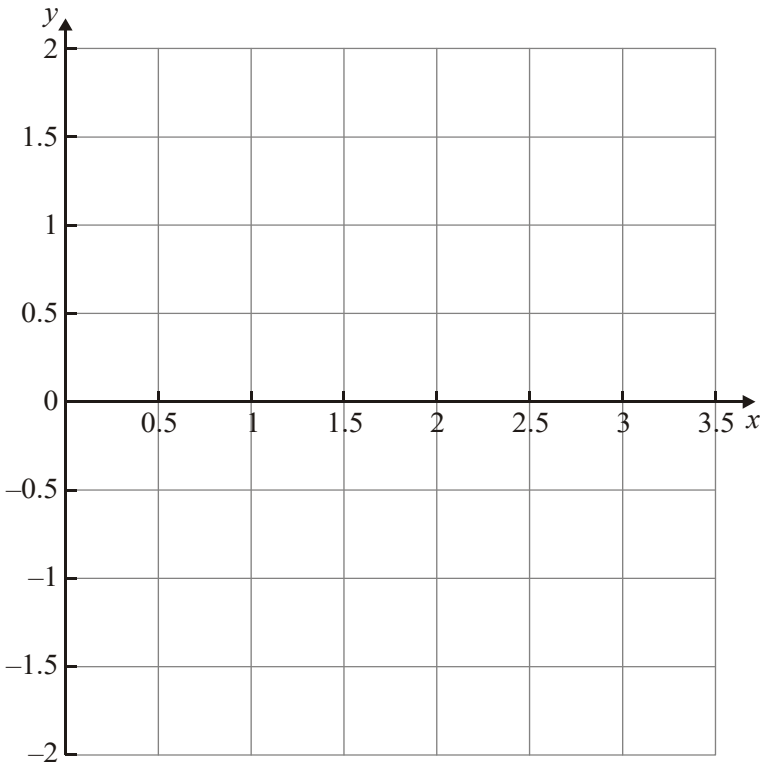
24. [Maximum mark: 6]

Let $f(x) = \sin(2x + 1)$, $0 \leq x \leq \pi$.

(a) Sketch the curve of $y = f(x)$ on the grid below. [2]



(b) Find the x-coordinates of the maximum and minimum points of $f(x)$, giving your answers correct to one decimal place. [4]

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25. [Maximum mark: 6]

Note: Radians are used throughout this question.

Let $f(x) = \sin(1 + \sin x)$.

(a) Write down the *x*-coordinates of all minimum and maximum points, for $0 \leq x \leq 6$. [4]

(b) Write down the range of $y = f(x)$, for $0 \leq x \leq 6$. [2]

Give your answers correct to **four** significant figures.

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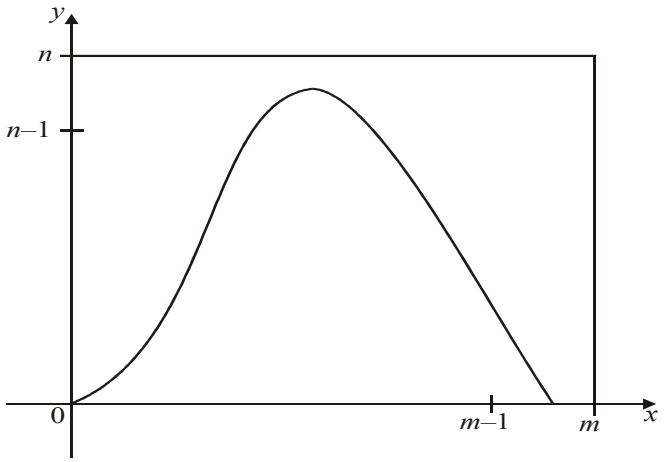
26. [Maximum mark: 4]

The diagram below shows the graph of $y = x \sin \frac{x}{3}$, for $0 \leq x < m$, and $0 \leq y < n$, where x is in radians and m and n are integers. Find the value of m and the value of n.



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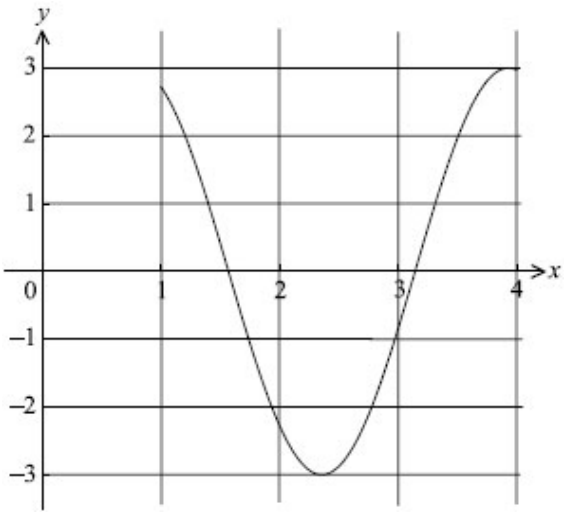
27. [Maximum mark: 6]

Let $f(x) = 3 \sin 2x$, for $1 \leq x \leq 4$. The graph of f is shown below.



(a) On the same diagram, sketch the graph of $g(x) = -5x^2 + 27x - 35$ for $1 \leq x \leq 4$. [2]

(b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution. [2]

(c) Let $h(x) = g(x) - f(x)$. Given that $h(x) \leq 0$ for $p \leq x \leq q$, write down the value of p and of q. [2]

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